1. A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

(a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet.
(1)

(b) Use this binomial distribution to find the probability that

(i) Aliya has just 1 red bead in her bracelet,

(ii) there are at least 4 red beads in Aliya's bracelet.
(3)

(c) Comment on the suitability of a binomial distribution to model this situation.
(1)

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.
(4)

(e) Find the *p*-value in this case.
(1)

(a) Let R = number of red beads in Aliya's bracelet

$R \sim B(18, 0.14)$ ⋇ (1)

(b)(i) $P(R=1) = 0.1940\ldots$ (1)

(ii) $P(R \geqslant 4) = 1 - P(R \leqslant 3)$

$= 1 - 0.761\ldots$ (1)

$= 0.238\ldots$ (1)

(c) Requires $p = 0.14$ to be constant so need large number of beads in the sack to ensure that removing 18 beads does not appreciably affect this probability, then it could be suitable. (1)

(d) Let X = Number of red beads in the sample

$X \sim B(75, 0.14)$ (1)

$E(X) = 75 \times 0.14 = 10.5$

$H_0 : p = 0.14$, $H_1 : p \neq 0.14$ (1)

$P(X \leqslant 4) = 0.01506 \ldots$ (1)

Since $0.01506\ldots < 0.025$, so we reject H_0.

$\therefore$ Hence, there is evidence to suggest proportion of red beads has changed. (1)

(e) p-value $= 2 \times 0.01506\ldots$

$= 0.030123$ (1)

2. In an experiment a group of children each repeatedly throw a dart at a target. For each child, the random variable H represents the number of times the dart hits the target in the first 10 throws.

Peta models H as B(10, 0.1)

(a) State two assumptions Peta needs to make to use her model.

(2)

(b) Using Peta's model, find $P(H \geqslant 4)$

(1)

For each child the random variable F represents the number of the throw on which the dart first hits the target.

Using Peta's assumptions about this experiment,

(c) find $P(F = 5)$

(2)

Thomas assumes that in this experiment no child will need more than 10 throws for the dart to hit the target for the first time. He models $P(F = n)$ as

$$P(F = n) = 0.01 + (n - 1) \times \alpha$$

where α is a constant.

(d) Find the value of α

(4)

(e) Using Thomas' model, find $P(F = 5)$

(1)

(f) Explain how Peta's and Thomas' models differ in describing the probability that a dart hits the target in this experiment.

(1)

a) • The probability of a dart hitting the target must be constant. - 1

• The throws of the dart are independent. - 1

b) $H \sim B(10, 0.1)$

$P(H \geqslant 4) = 1 - P(H \leq 3)$

$= 1 - 0.9872$

$= 0.0128$ (4d.p.)

$P(H \geqslant 4) = 0.0128$ (4d.p.) - 1

c) $1 - 0.1 = 0.9$ ← probability of missing the target

$P(F=5) = 0.9^4 \times 0.1$ - 1

$= 0.0656$ (4d.p.) - 1

d)

n	1	2	...	10
$P(F=n)$	0.01	$0.01+\alpha$	...	$0.01+9\alpha$

- 1

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(0.01) + 9\alpha]$

$\therefore\ 5(0.02 + 9\alpha) = 1$ - 2

$0.1 + 45\alpha = 1$

$45\alpha = 0.9$

$\alpha = 0.02$ - 1

e) $P(F=5) = 0.01 + 4(0.02)$

$= 0.09$ - 1

f) Peta's model assumes probability of hitting a target is constant, whereas Thomas' model assumes the probability increases with each throw. - 1

3. Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(a) Find the probability that it has a daily mean total cloud cover of 6 or greater.

$52+52+28=132$ $\quad 132/184 = \frac{33}{46}$ (1)

(1)

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable X to denote the daily mean total cloud cover and believes that $X \sim B(8, 0.76)$

n p

Using Magali's model, $P(X \leq x)$ ← Form needed for calculator (1)

4 5 | 6 7 8

(b) (i) find $P(X \geqslant 6)$ (1)

$P(X \geqslant 6) = 1 - P(X \leq 5) = 1 - 0.296722... = 0.703$ (3dp)

(2)

(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7

$P(X=7) \times 184 = 0.2811... \times 184 = 51.7385... = 51.7$ (1dp) (1) (1)

(2)

(c) Explain whether or not your answers to part (b) support the use of Magali's model.

(1)

Part (a) and part (b)(i) are similar and the expected number of 7s (51.7) matches number of 7s in the data set (52) so Magali's model is supported (1)

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(d) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.

(1)

$5+9+9=23$ $\quad \frac{23}{28} = 0.82142... = 0.821$ (3dp) (1)

(e) Comment on Magali's model in light of your answer to part (d).

(2)

Part d (0.821) differs from part (a) and (b)(i) (≈ 0.7) (1)
therefore Magali's model may not be suitable (1)
(since this means independence does not hold)

4. (a) State one disadvantage of using quota sampling compared with simple random sampling.

(1)

In a university 8% of students are members of the university dance club. so $p = 0.08$

A random sample of 36 students is taken from the university. and $n = 36$

The random variable X represents the number of these students who are members of the dance club.

(b) Using a suitable model for X, find

(i) $P(X = 4)$

(ii) $P(X \geqslant 7)$

(3)

Only 40% of the university dance club members can dance the tango.

(c) Find the probability that a student is a member of the university dance club and can dance the tango.

use multiplication rule for AND

(1)

A random sample of 50 students is taken from the university.

$P(X < 3)$

(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.

(2)

a) One disadvantage is that quota sampling is not random, so it cannot be used reliably for inferences. 1

OR

More likely to be biased.

OR

Not random / less random

b) i) $X \sim B(36, 0.08)$ 1 where $n = 36$, $p = 0.08$

$P(X = 4) = 0.16738... = 0.167$ (3.s.f) 1

ii) $P(X \geqslant 7) = 1 - P(X \leqslant 6)$

$= 1 - 0.97776..$

$= 0.022233..$

$= 0.0222$ (3 s.f) 1

c) $P(\text{dance club AND tango}) = 0.08 \times 0.4$

$= 0.032$ 1

or 3.2% or $\frac{4}{125}$

d) T = people who can dance the tango

$T \sim B(50, 0.032)$ 1 0.032 from c)

$P(T < 3) = P(T \leq 2) = 0.785$ (3.s.f) 1

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